Problem of the week

Gravitational fields (SL)

(a) The gravitational field strength on the surface of earth is g. What is the gravitational field strength

(i) at a distance of two earth radii from the **surface** of earth,

(ii) on the surface of a planet X that has double the mass and double the radius of earth,

(iii) on the surface of a planet Y that has the same density as earth and double the radius of earth.

(b) In projectile motion we assume that the gravitational field above the earth's surface is uniform.

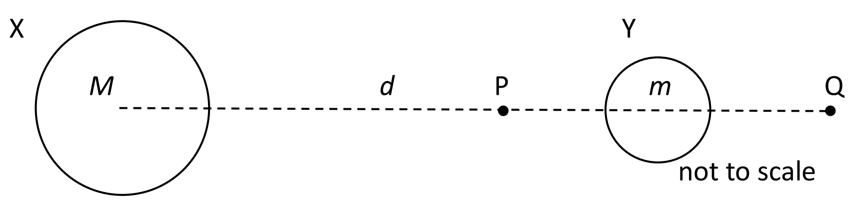
(i) Draw the gravitational field lines in this case.

(ii) Draw a realistic set of gravitational field lines for earth.

(iii) State what feature of the diagram in (ii) allows us to deduce that g decreases as we move away from the surface.

(iv) Suggest why two gravitational field lines cannot cross.

(c) The diagram shows two planets, X and Y, of the same density and mass M and m. The planets are a distance d apart center to center.

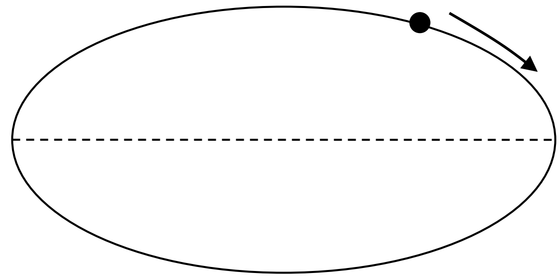


(i) At point P, a distance $0.75d$ from the center of X, the gravitational field strength is zero. Determine the ratio $\frac{M}{m}$.

(ii) For the ratio in (i) determine the gravitational field strength at point Q, a distance of $0.25d$ to the right of the center of Y given that $\frac{GM}{d^2} = 2.0\ \text{N kg}^{-1}$.

(iii) Draw a graph (no numbers required) to show the variation of the resultant gravitational field with distance along the dotted line from the surface of X to the surface of Y. Take the positive direction to be that to the right.

(d) The diagram shows the elliptical orbit of a planet around the Sun. As the planet approaches point X, the speed increases.



(i) Draw the approximate position of the Sun,

(ii) Explain your answer to (i).

(e) A planet is in a circular orbit of radius R around a star of mass M.

(i) Show that the period of revolution of the planet is given by $T^2 = \frac{4\pi^2}{GM}R^3$.

(ii) Io is a moon of Jupiter with an orbital radius of 4.2×10^8 m and an orbital period of 42 hours. Estimate the period of the Jupiter moon Callisto whose orbital radius is 1.9×10^9 m.

Answers

(a)

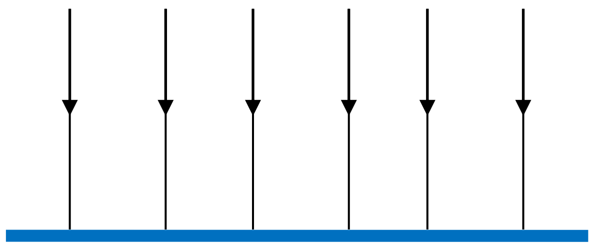
(i) $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(3R)^2} = \frac{1}{9}\frac{GM}{R^2} = \frac{g}{9}$.

(ii) $g' = \frac{G(2M)}{(2R)^2} = \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2}$.

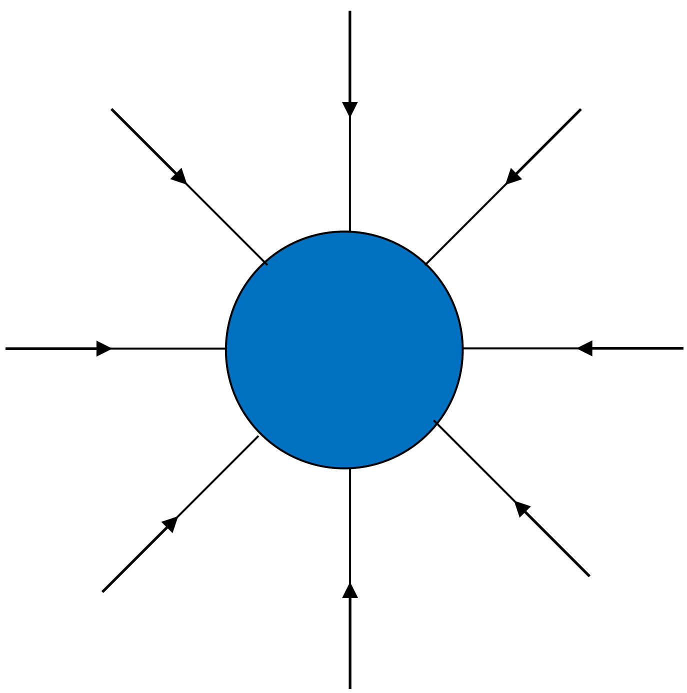
(iii) The volume is 8 times larger, so the mass is 8 times larger. Hence $g' = \frac{G(8M)}{(2R)^2} = 2\frac{GM}{R^2} = 2g$.

(b)

(i)



(ii)



(iii) The field lines are getting further apart/their density is decreasing.

(iv) Tangents to field lines give the direction of the gravitational field which is unique at a point. Crossing lines would give two directions.

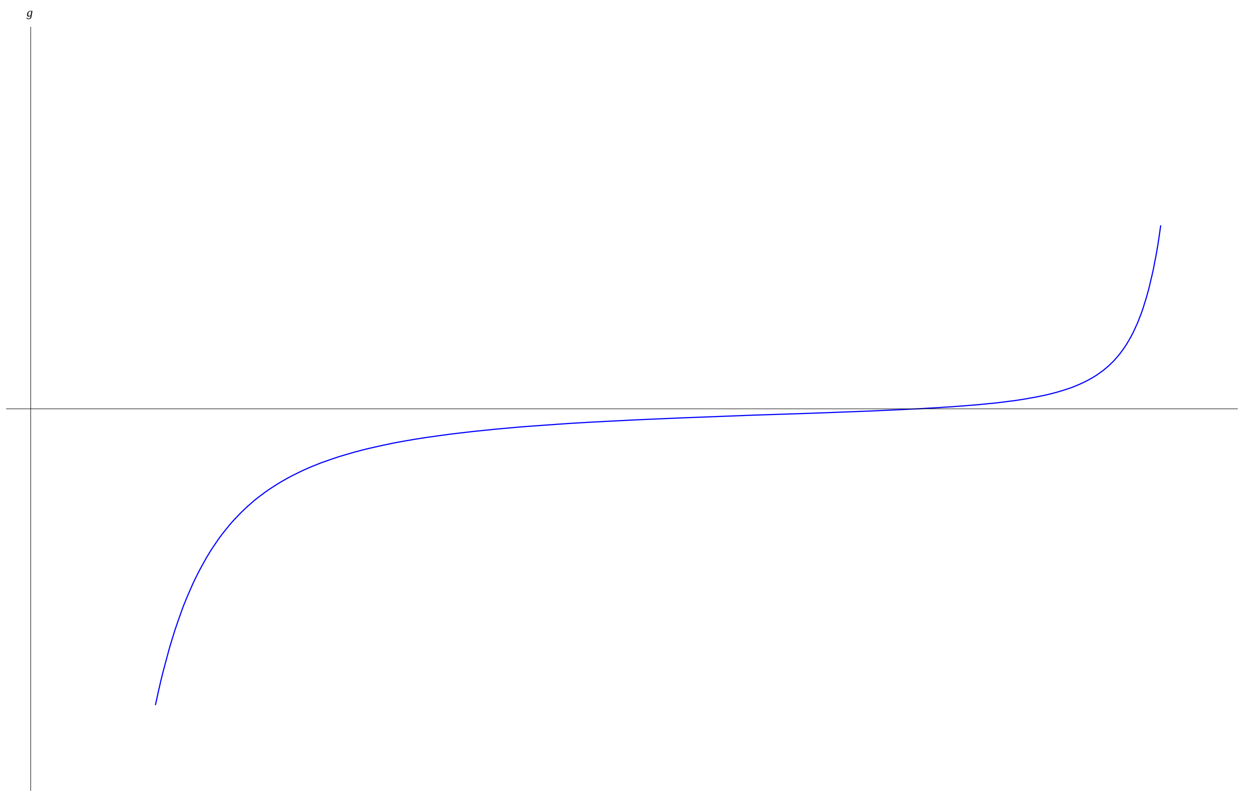
(c)

(i) The net field is zero so $\frac{GM}{(0.75d)^2} = \frac{Gm}{(0.25d)^2} \Rightarrow \frac{M}{m} = (\frac{0.75}{0.25})^2 = 9$.

(ii) The net field at Q is

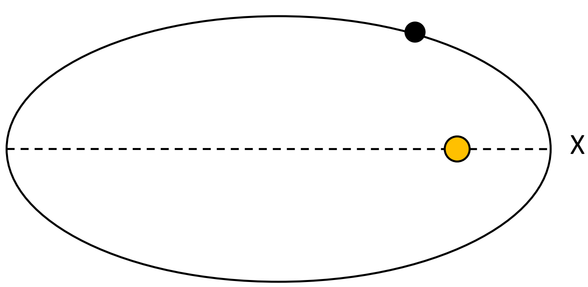
$$\frac{GM}{(1.25d)^2} + \frac{Gm}{(0.25d)^2} = \frac{GM}{(1.25d)^2} + \frac{GM}{9\times(0.25d)^2} = \frac{544GM}{225d^2} = \frac{544}{225}\times 2.0 = 4.8\ \text{N kg}^{-1}.$$

(iii) Something like: (we care only about the general shape here)



(d)

(i)



(ii) There is a component of the gravitational force in the direction of velocity, so speed is increasing.

(e)

(i) $\frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow v^2 = \frac{GM}{R}$,

$v = \frac{2\pi R}{T} \Rightarrow (\frac{2\pi R}{T})^2 = \frac{GM}{R}$ hence, $T^2 = \frac{4\pi^2}{GM}R^3$.

(ii) $\frac{T_{\text{Callisto}}}{T_{\text{Io}}} = \left(\frac{R_{\text{Callisto}}}{R_{\text{Io}}}\right)^{\frac{3}{2}} = \left(\frac{1.9\times10^9}{4.2\times10^8}\right)^{\frac{3}{2}} = 96.2$. Hence

$T_{\text{Callisto}} = 96.2\times42 = 4.04\times10^3 \text{ hr} = 16.8 \approx 17 \text{ days}$